

The Essence of C1



Quadratics

$$ax^2 + bx + c = 0 \text{ has roots } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $b^2 - 4ac > 0 \Rightarrow 2 \text{ real roots}$

$$b^2 - 4ac = 0 \Rightarrow 1 \text{ real root}$$

$$b^2 - 4ac < 0 \Rightarrow \text{no real roots}$$

Completing the square: $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

Difference of two squares: $(x - y)(x + y) \equiv x^2 - y^2$

Indices

$$a^m \times a^n = a^{m+n} \quad (a^m)^n = a^{mn} \quad a^0 = 1$$

$$a^m \div a^n = a^{m-n} \quad a^{-m} = \frac{1}{a^m} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Surds

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

To rationalise the denominator of $\frac{1}{a \pm \sqrt{b}}$

multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$

Equations & Inequalities

- To solve sim. eqns with one linear and one quadratic, substitute linear into quadratic.
- To solve quadratic inequality, find roots and think about the graph.
- If an inequality is multiplied or divided by a -ve number, remember to flip the sign.

Coordinate Geometry

Gradient of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a line if you know:

- gradient and the intercept $\Rightarrow y = mx + c$
- gradient and one point $\Rightarrow y - y_1 = m(x - x_1)$
- two points $\Rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

Gradient of normal is $-\frac{1}{m}$

Sequences and Series

A *sequence* is a list of numbers following a rule. A *series* is the sum of a sequence.
Recurrence relationship: the next term is some function of the current one.

$$u_{n+1} = f(u_n), n \geq 1$$

Arithmetic sequence: the difference between consecutive terms is constant.

$$a, a + d, a + 2d, a + 3d, \dots$$

Learn the proof of the formula for S_n

© n^{th} term: $u_n = a + (n - 1)d$

© Sum of n terms: $S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}\{a + L\}$

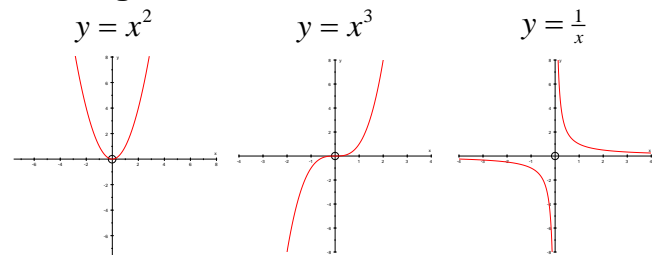
Integration

$$y = x^n \Rightarrow \int y \, dx = \frac{x^{n+1}}{n+1} + C$$

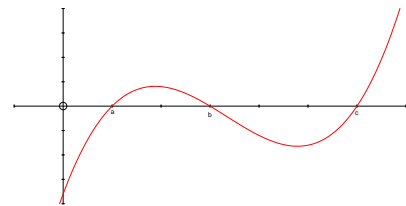
Do NOT forget the constant of integration.

Given a point on the curve, C can be found.

Sketching Curves



$$y = (x - a)(x - b)(x - c)$$



Transformations of $f(x)$ with $a > 0$,

- $f(x + a)$ is a shift left
- $f(x - a)$ is a shift right
- $af(x)$ is a vertical stretch of factor a
- $f(ax)$ is a horizontal squeeze
- $-f(x)$ is a reflection in the x -axis
- $f(-x)$ is a reflection in the y -axis

Differentiation

The gradient of a function $y = f(x)$ is $\frac{dy}{dx}$ and is sometimes written $f'(x)$ and it is called the derivative of the function.

$$y = kx^n \Rightarrow \frac{dy}{dx} = f'(x) = knx^{n-1}$$

$$\frac{d(f(x) \pm g(x))}{dx} = \frac{d(f(x))}{dx} \pm \frac{d(g(x))}{dx}$$

Eqn of tangent at (a, b) : $y - b = f'(a)(x - a)$

Eqn of normal at (a, b) : $y - b = \frac{-1}{f'(a)}(x - a)$