The Essence of C2

Algebra for a polynomial $f(x)$

Remainder Theorem:

If $f(x)$ is divided by $(x-a)$ the remainder is $f(a)$

If $f(x)$ is divided by $(ax-b)$ the remainder is $f(\frac{b}{a})$

Factor Theorem:

If $f(a)=0$, then $(x-a)$ is a factor of $f(x)$

If $f\left(\frac{b}{a}\right)=0$, then $(ax-b)$ is a factor of $f(x)$

Exponentials and Logarithms

$y = a^x \Leftrightarrow \log_a y = x$

Rule 1 $\log_a xy = \log_a x + \log_a y$

Rule 2 $\log_a \frac{x}{y} = \log_a x - \log_a y$

Rule 3 $\log_a x^k = k \log_a x$

Rule 4 $\log_a \frac{1}{x} = -\log_a x$

Rule 5 $\log_a a = 1, \log_a 1 = 0$

A Useful One: $\log_a b = \frac{1}{\log_b a}$

☺ Change of Base: $\log_a x = \frac{\log_b x}{\log_b a}$

Coordinate Geometry

Midpoint of a line: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Dist. between 2 points: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Equation of circle, centre O, radius r: $x^2+y^2=r^2$

Equation of circle, centre (a,b), radius r: $(x-a)^2+(y-b)^2=r^2$

Sequences and Series

Geometric Series: the ratio of consecutive terms is constant.

$a, ar, ar^2, ar^3, ar^4, \ldots$

☺ n^{th} term: ar^{n-1}

☺ Sum of n terms: $S_n = \frac{a(1-r^n)}{1-r}$

☺ If $|r|<1$ $S_\infty = \frac{a}{1-r}$

Learn the proof of the formula for S_n

Binomial Expansion

$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$

☺ ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ $0! = 1$

☺ $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}b^n$

☺ $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n$

Trigonometry

For a circle: (θ in radians)

Arc length, $s = r\theta$ Sector area, $A = \frac{1}{2}r^2\theta$

Degrees $\xrightarrow{\times\frac{\pi}{180}}$ Radians Radians $\xrightarrow{\times\frac{180}{\pi}}$ Degrees

Learn the following basic results:

deg	rad	$sin\theta$	$cos\theta$	$tan\theta$
0	0	0	1	0
30	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\pi/4$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\pi/2$	1	0	∞

Sin | All

Tan | Cos

Solve equations using:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

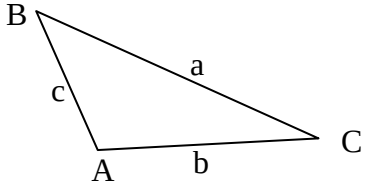
$\sin^2\theta + \cos^2\theta = 1$

You must be able to sketch the graphs of $sin\theta, cos\theta$ and $tan\theta$

Area of Triangle: $\frac{1}{2}ab\sin\theta$

Sine Rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

☺ Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$



Differentiation

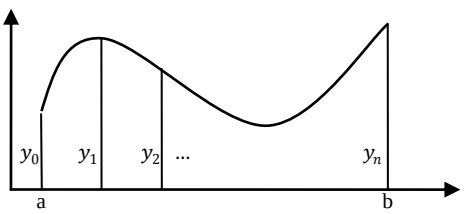
Stationary points at $f'(x) = 0$, then check the second derivative.

$f''(x)\begin{cases} > 0 \Rightarrow minimum \\ < 0 \Rightarrow maximum \\ = 0 \Rightarrow point\ of\ inflection \end{cases}\Big\} turning\ points$

Know the terms 'increasing' and 'decreasing'

Trapezium Rule

☺ $\int_a^b y\ dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\}$, where $h = \frac{b-a}{n}$



Definite Integration

$\int_a^b f'(x)\ dx = [f(x)]_a^b = f(b) - f(a)$

☺ = given in the Formulae Booklet