The Essence of C3

Algebraic Fractions

- Simplify by cancelling factors, so factorise if possible
- To multiply, simplify first if possible then multiply the numerators and the denominators
- To divide, reciprocate the second fraction and multiply
- To add or subtract, a common denominator is needed
  Use the **lowest** common denominator for simplicity
- If the numerator has the same or higher degree (order) as the numerator, then you may need to do a long division or use the remainder theorem

  eg

$$\begin{array}{r|l} & x^2 + \ 4x + 12 \\ \hline x-3 & x^3 + \ x^2 + \ 0x - \ 7 \\ & \underline{x^3 - 3x^2} \\ & \qquad 4x^2 + \ 0x \\ & \qquad \underline{4x^2 - 12x} \\ & \qquad\qquad 12x - \ 7 \\ & \qquad\qquad \underline{12x - 36} \\ & \qquad\qquad\qquad 29 \end{array}$$

So $\frac{x^2+4x+12}{x-3} = x^2 + 4x + 12 + \frac{29}{x-3}$

Functions

- A function is a mapping in which each member of the *domain* is mapped to one member of the *range*
- Functions can be *many-to-one* or *one-to-one*
- Functions can combine to form *composite* functions

  eg If $f(x) = 3x - 1$ and $g(x) = x^2$, then $fg(2)$ means 'do f to the answer to $g(2)$'
  So $fg(2) = f(4) = 11$
  Similarly $fg(x) = f(x^2) = 3x^2 - 1$
  and $gf(x) = g(3x - 1) = (3x - 1)^2$
- The inverse of a function $f(x)$ is written $f^{-1}(x)$ and performs the opposite operation to $f(x)$
- eg If $h(x) = 4x - 5$, then $h^{-1}(x) = \frac{x+5}{4}$
- The range of the function is the domain of the inverse function and vice versa
- The graph of $f^{-1}(x)$ is a reflection of $f(x)$ in the line $y = x$

Exponential and Log Functions

- Exponential functions are of the form $y = a^x$
- $y = e^x$ is called **the** exponential function (where $e \approx 2.718$)
- The inverse of e^x is $\ln x$
- You must be able to sketch $y = e^x$ and $y = \ln x$
- You must be able to solve equations with e^x and $\ln x$
- Growth and decay models are based around the exponential function

Numerical Methods

- Find an interval in which the sign of $f(x)$ changes
  There will be a root of $f(x) = 0$ in that interval, assuming $f(x)$ is continuous
- To solve $f(x) = 0$, rearrange into a form $x = g(x)$
  Use the iterative formula $x_{n+1} = g(x_n)$
- For different rearrangements, the formula may converge to a root or may diverge
- For different initial values, x_0, the formula may converge to a root or may diverge
- If a question asks you to show that α is a root to a certain degree of accuracy, you should find $f(lower\ bound)$ and $f(upper\ bound)$ and comment that the sign changes

Transforming Graphs

- The modulus of a number a, written $|a|$, is its positive numerical value
- To sketch $y = |f(x)|$, imagine $y = f(x)$ and reflect any section below the x-axis in the x-axis
- To sketch $y = f(|x|)$, imagine $y = f(x)$ and erase the part where x<0. Reflect the remainder in the y-axis
  NB If $f(x) = x^3 - 9x$, then $f(|x|) = |x|^3 - 9|x|$
- You must be able to solve equations involving modulus. Sketching the graphs will help greatly
- Remember from C1
  Transformations of $f(x)$ with $a > 0$,
  $f(x + a)$ is a shift left
  $f(x - a)$ is a shift right by a
  $a\,f(x)$ is a vertical stretch of factor a
  $f(ax)$ is a horizontal squeeze of factor $1/a$
  $-f(x)$ is a reflection in the x-axis
  $f(-x)$ is a reflection in the y-axis
- When sketching a graph, give the points where it hits the axes where possible. Also give the new position of any point given on the original curve.

Trigonometry 1

- $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$
- You must be able to sketch $\sec x$, $\operatorname{cosec} x$ and $\cot x$
- Three vital identities:
  $\sin^2\theta + \cos^2\theta = 1$
  $\tan^2\theta + 1 = \sec^2\theta$
  $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- You must be able to solve equations and prove identities containing any of these functions
- You must be able to sketch the inverse functions

Trigonometry 2

- Addition Formulae
  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
  $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- Double Angle Formulae
  $\sin 2A = 2\sin A \cos A$
  $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$
  $\cos 2A = \cos^2 A - \sin^2 A$
  $= 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$
- You will be asked about one of these
  $a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$
  $a\cos\theta \pm b\sin\theta = R\cos(\theta \pm \alpha)$
- Factor Formulae

$$\sin P + \sin Q = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

$$\cos P + \cos Q = 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

- A variation of these can be useful
  $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
  $2\cos A\sin B = \sin(A + B) - \sin(A - B)$
  $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
  $2\sin A\sin B = -\{\cos(A + B) - \cos(A - B)\}$

Differentiation

- Chain Rule

  If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- Product Rule

  If $y = u.v$, then $\frac{dy}{dx} = v.\frac{du}{dx} + u.\frac{dv}{dx}$
- Quotient Rule

  If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Standard Results (to be combined with these rules)

y	$\frac{dy}{dx}$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$